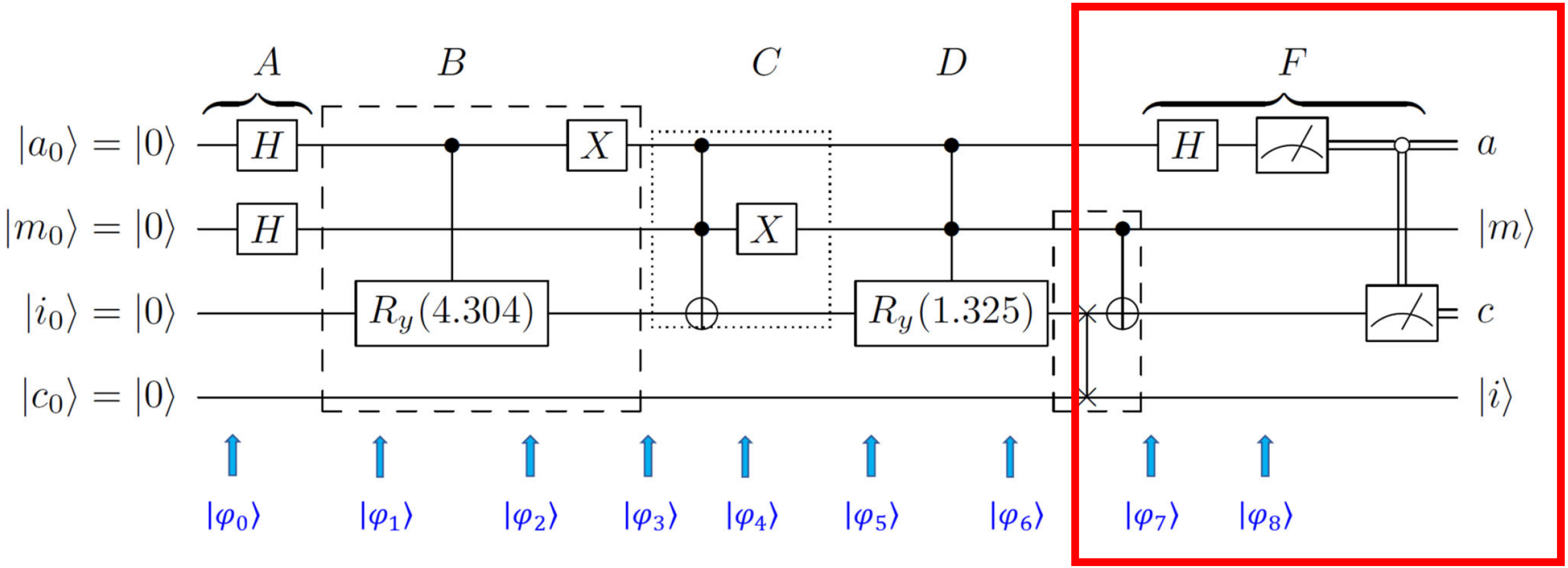
Application of the Hadamard Gate to Get to the Probability of $1\text{-}d^2$



A
B
C
D
F
$|a_0\rangle = |0\rangle$
$|m_0\rangle = |0\rangle$
$|i_0\rangle = |0\rangle$
$|c_0\rangle = |0\rangle$
H
H
X
X
H
$R_y(4.304)$
$R_y(1.325)$
a
$|m\rangle$
c
$|i\rangle$
$|\varphi_0\rangle$
$|\varphi_1\rangle$
$|\varphi_2\rangle$
$|\varphi_3\rangle$
$|\varphi_4\rangle$
$|\varphi_5\rangle$
$|\varphi_6\rangle$
$|\varphi_7\rangle$
$|\varphi_8\rangle$

Titanic Example: Nearest Neighbour Method

The point of this exercise is to determine if an unlabeled passenger survived or died from the sinking of the Titanic.

Labels and Classifiers

Each vector represents a passenger who was on the Titanic with two specific features: ticket price (0) and cabin number (1)

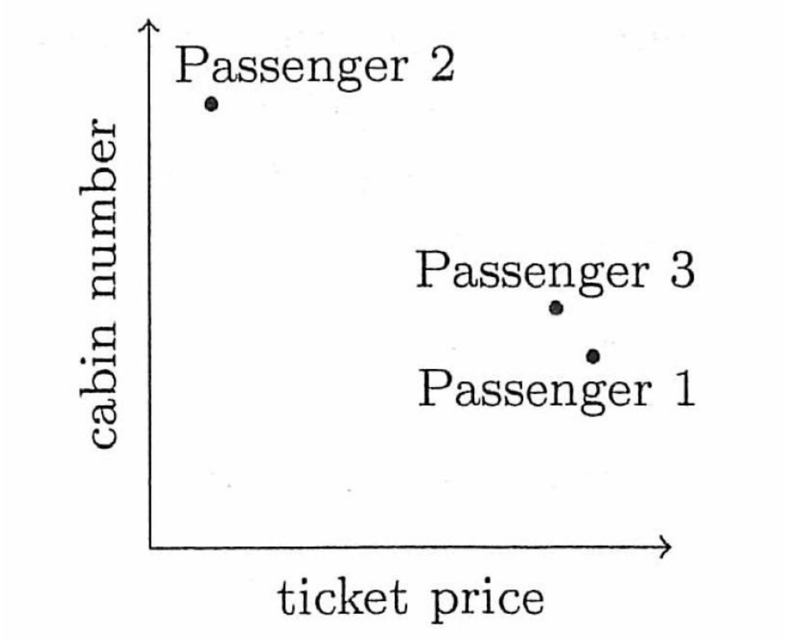
- Ticket prices between $0 to $10,000
- Cabin number range between 1 to 2,500

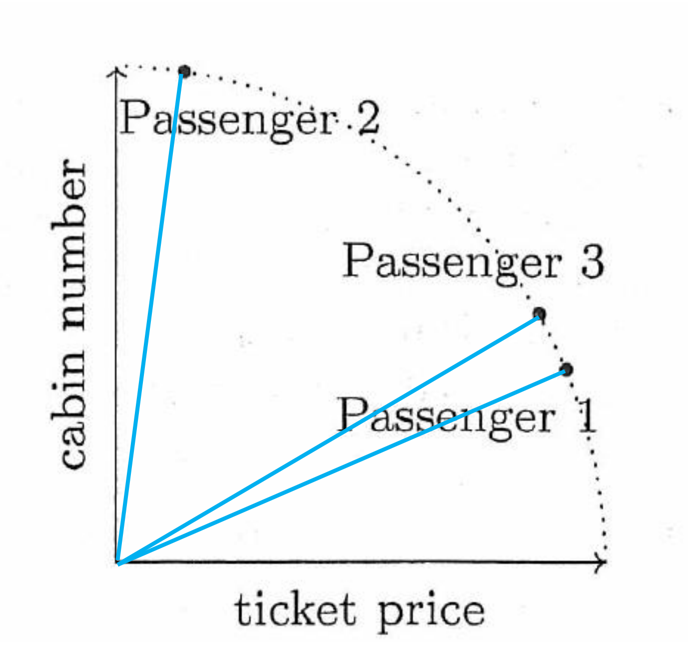
Each input vector, x^m assigned a label y^m to indicate if survived (y^m = 1) or died (y^m = 0)

Step A: Preprocessing by Normalizing the Data

Next step is to normalize the points on the graph to one to transform onto a unit circle

- Now each point has information about the angles between the data vectors





On the unit circle P_3 is still closest to P_1

Determining Probability: Squared Distance Classifier

Know: Passenger 3 (P_3) is closer to Passenger 1 (P_1) than Passenger 2 (P_2)
Now: Need to determine the probability of P_3 living or dying

Probability Equation

Probability of $\boldsymbol{y}$, survived or died, given the new input $\boldsymbol{x}$ as the sum over the weights of all M_1 training inputs which are labeled $\boldsymbol{y^m = 1}$

Probability of predicting label 1: $p(y = 1|x) = \frac{1}{\chi}\frac{1}{M_1}\sum_{m|y^m=1}(1 - \frac{1}{c}||x - x^m||^2)$

Probability of predicting label 0: $p(y = 0|x) = \frac{1}{\chi}\frac{1}{M_1}\sum_{m|y^m=0}(1 - \frac{1}{c}||x - x^m||^2)$

$\frac{1}{X}$ *included to make sure* $p(y = 0|x) + p(y = 1|x) = 1$

Data Encoding

The data is encoded in a quantum system in order to us the Hadamard transform.

q_1: 0 = P_1 or P_2
1 = P_3

q_2: 0 = P_1
1 = P_2

q_3: 0 = Price
1 = Cabin

q_4: 0 = Died
1 = Survived

Passenger 1 or 2 (rows with $q_1 = 0$); Passenger 3 (rows with $q_1 = 1$)

Qubit state				Transformation of amplitude vector		
q_1	q_2	q_3	q_4	Step B α_{init}	Step C α_{inter}	Step D α_{final}
0	0	0	0	0	0	0
0	0	0	1	$\frac{1}{\sqrt{4}}0.921$	$\frac{1}{\sqrt{8}}(0.921 + 0.866)$	$\frac{1}{\sqrt{8\chi}}(0.921 + 0.866)$
0	0	1	0	0	0	0
0	0	1	1	$\frac{1}{\sqrt{4}}0.390$	$\frac{1}{\sqrt{8}}(0.390 + 0.500)$	$\frac{1}{\sqrt{8\chi}}(0.390 + 0.500)$
0	1	0	0	$\frac{1}{\sqrt{4}}0.141$	$\frac{1}{\sqrt{8}}(0.141 + 0.866)$	$\frac{1}{\sqrt{8\chi}}(0.141 + 0.866)$
0	1	0	1	0	0	0
0	1	1	0	$\frac{1}{\sqrt{4}}0.990$	$\frac{1}{\sqrt{8}}(0.990 + 0.500)$	$\frac{1}{\sqrt{8\chi}}(0.990 + 0.500)$
0	1	1	1	0	0	0
1	0	0	0	0	0	0
1	0	0	1	$\frac{1}{\sqrt{4}}0.866$	$\frac{1}{\sqrt{8}}(0.921 - 0.866)$	0
1	0	1	0	0	0	0
1	0	1	1	$\frac{1}{\sqrt{4}}0.500$	$\frac{1}{\sqrt{8}}(0.390 - 0.500)$	0
1	1	0	0	$\frac{1}{\sqrt{4}}0.866$	$\frac{1}{\sqrt{8}}(0.141 - 0.866)$	0
1	1	0	1	0	0	0
1	1	1	0	$\frac{1}{\sqrt{4}}0.500$	$\frac{1}{\sqrt{8}}(0.990 - 0.500)$	0
1	1	1	1	0	0	0

* P1 = Passenger 1 P2 = Passenger 2 P3 = Passenger 3

Step B: Data Encoding

Example:

- $q_1 = 0 = P_1$ or P_2
- $q_2 = 0 = P_1$
- $q_3 = 0 =$ Price
- $q_4 = 1 =$ Survived

Use the value 0.921 from the table below

	price	room	survival
Passenger 1	0.921	0.390	yes (1)
Passenger 2	0.141	0.990	no (0)
Passenger 3	0.866	0.500	?

Qubit state				Transformation of amplitude vector		
q_1	q_2	q_3	q_4	Step B α_{init}	Step C α_{inter}	Step D α_{final}
0	0	0	0	0	0	0
0	0	0	1	$\frac{1}{\sqrt{4}}0.921$	$\frac{1}{\sqrt{8}}(0.921 + 0.866)$	$\frac{1}{\sqrt{8\chi}}(0.921 + 0.866)$
0	0	1	0	0	0	0
0	0	1	1	$\frac{1}{\sqrt{4}}0.390$	$\frac{1}{\sqrt{8}}(0.390 + 0.500)$	$\frac{1}{\sqrt{8\chi}}(0.390 + 0.500)$
0	1	0	0	$\frac{1}{\sqrt{4}}0.141$	$\frac{1}{\sqrt{8}}(0.141 + 0.866)$	$\frac{1}{\sqrt{8\chi}}(0.141 + 0.866)$
0	1	0	1	0	0	0
0	1	1	0	$\frac{1}{\sqrt{4}}0.990$	$\frac{1}{\sqrt{8}}(0.990 + 0.500)$	$\frac{1}{\sqrt{8\chi}}(0.990 + 0.500)$
0	1	1	1	0	0	0
1	0	0	0	0	0	0
1	0	0	1	$\frac{1}{\sqrt{4}}0.866$	$\frac{1}{\sqrt{8}}(0.921 - 0.866)$	0
1	0	1	0	0	0	0
1	0	1	1	$\frac{1}{\sqrt{4}}0.500$	$\frac{1}{\sqrt{8}}(0.390 - 0.500)$	0
1	1	0	0	$\frac{1}{\sqrt{4}}0.866$	$\frac{1}{\sqrt{8}}(0.141 - 0.866)$	0
1	1	0	1	0	0	0
1	1	1	0	$\frac{1}{\sqrt{4}}0.500$	$\frac{1}{\sqrt{8}}(0.990 - 0.500)$	0
1	1	1	1	0	0	0

Step C: Hadamard Transformation

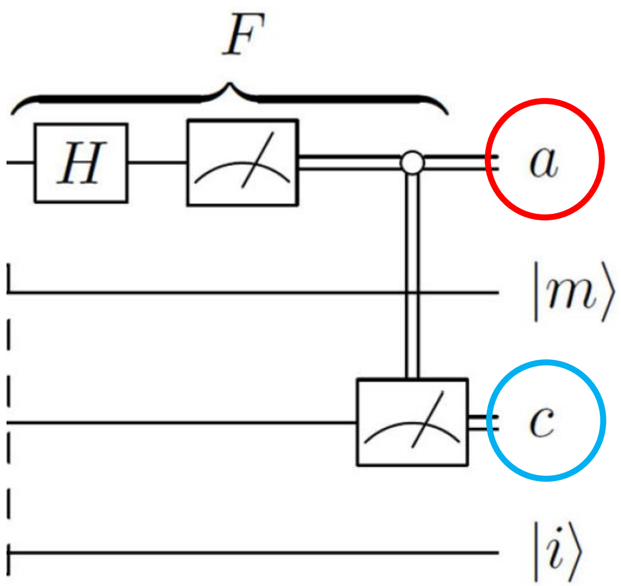
Apply Hadamard matrix to Step B

$$\alpha \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \alpha \begin{bmatrix} a+b \\ a-b \end{bmatrix}$$

Qubit state				Transformation of amplitude vector		
q_1	q_2	q_3	q_4	Step B α_{init}	Step C α_{inter}	Step D α_{final}
0	0	0	0	0	0	0
0	0	0	1	$\frac{1}{\sqrt{4}}0.921$	$\frac{1}{\sqrt{8}}(0.921 + 0.866)$	$\frac{1}{\sqrt{8\chi}}(0.921 + 0.866)$
0	0	1	0	0	0	0
0	0	1	1	$\frac{1}{\sqrt{4}}0.390$	$\frac{1}{\sqrt{8}}(0.390 + 0.500)$	$\frac{1}{\sqrt{8\chi}}(0.390 + 0.500)$
0	1	0	0	$\frac{1}{\sqrt{4}}0.141$	$\frac{1}{\sqrt{8}}(0.141 + 0.866)$	$\frac{1}{\sqrt{8\chi}}(0.141 + 0.866)$
0	1	0	1	0	0	0
0	1	1	0	$\frac{1}{\sqrt{4}}0.990$	$\frac{1}{\sqrt{8}}(0.990 + 0.500)$	$\frac{1}{\sqrt{8\chi}}(0.990 + 0.500)$
0	1	1	1	0	0	0
1	0	0	0	0	0	0
1	0	0	1	$\frac{1}{\sqrt{4}}0.866$	$\frac{1}{\sqrt{8}}(0.921 - 0.866)$	0
1	0	1	0	0	0	0
1	0	1	1	$\frac{1}{\sqrt{4}}0.500$	$\frac{1}{\sqrt{8}}(0.390 - 0.500)$	0
1	1	0	0	$\frac{1}{\sqrt{4}}0.866$	$\frac{1}{\sqrt{8}}(0.141 - 0.866)$	0
1	1	0	1	0	0	0
1	1	1	0	$\frac{1}{\sqrt{4}}0.500$	$\frac{1}{\sqrt{8}}(0.990 - 0.500)$	0
1	1	1	1	0	0	0

Step D: Measuring the First Qubit and Conditionally Measure the Third Qubit

If q_1 is found in the state 0 (a), then measure the label qubit (c) otherwise trash it.



Qubit state				Transformation of amplitude vector		
q_1	q_2	q_3	q_4	Step B α_{init}	Step C α_{inter}	Step D α_{final}
0	0	0	0	0	0	0
0	0	0	1	$\frac{1}{\sqrt{4}}0.921$	$\frac{1}{\sqrt{8}}(0.921+0.866)$	$\frac{1}{\sqrt{8\chi}}(0.921+0.866)$
0	0	1	0	0	0	0
0	0	1	1	$\frac{1}{\sqrt{4}}0.390$	$\frac{1}{\sqrt{8}}(0.390+0.500)$	$\frac{1}{\sqrt{8\chi}}(0.390+0.500)$
0	1	0	0	$\frac{1}{\sqrt{4}}0.141$	$\frac{1}{\sqrt{8}}(0.141+0.866)$	$\frac{1}{\sqrt{8\chi}}(0.141+0.866)$
0	1	0	1	0	0	0
0	1	1	0	$\frac{1}{\sqrt{4}}0.990$	$\frac{1}{\sqrt{8}}(0.990+0.500)$	$\frac{1}{\sqrt{8\chi}}(0.990+0.500)$
0	1	1	1	0	0	0
1	0	0	0	0	0	0
1	0	0	1	$\frac{1}{\sqrt{4}}0.866$	$\frac{1}{\sqrt{8}}(0.921-0.866)$	0
1	0	1	0	0	0	0
1	0	1	1	$\frac{1}{\sqrt{4}}0.500$	$\frac{1}{\sqrt{8}}(0.390-0.500)$	0
1	1	0	0	$\frac{1}{\sqrt{4}}0.866$	$\frac{1}{\sqrt{8}}(0.141-0.866)$	0
1	1	0	1	0	0	0
1	1	1	0	$\frac{1}{\sqrt{4}}0.500$	$\frac{1}{\sqrt{8}}(0.990-0.500)$	0
1	1	1	1	0	0	0

Renormalization Factor:

$$\chi = \frac{1}{8}\left(|0.921+0.866|^2 + |0.390+0.500|^2 + |0.141+0.866|^2 + |0.990+0.500|^2\right.$$

$$= 0.902$$

Step E: Measuring the Last Qubit

$$p(y=0|x) = \frac{1}{\chi}\frac{1}{M_1}\sum_{m|y^m=1}\left(1-\frac{1}{c}||x-x^m||^2\right)$$

$$p(q_4=0) = \frac{1}{8\chi}\left(1-\frac{1}{4}\left(|0.141+0.866|^2+|0.990+0.500|^2\right)\right)$$
$$= 0.448$$

χ is to measure the top half of the column vector: top 8 states

- $\chi = 4$ because should only be four numerical outputs

Step D α_{final}
0
$\frac{1}{\sqrt{8\chi}}(0.921+0.866)$
0
$\frac{1}{\sqrt{8\chi}}(0.390+0.500)$
$\frac{1}{\sqrt{8\chi}}(0.141+0.866)$
0
$\frac{1}{\sqrt{8\chi}}(0.990+0.500)$
0
0
0
0
0
0
0
0
0

Step E: Measuring the Last Qubit

$$p(y=1|x) = \frac{1}{\chi}\frac{1}{M_1}\sum_{m|y^m=1}\left(1-\frac{1}{c}||x-x^m||^2\right)$$

$$p(q_4=1) = \frac{1}{8\chi}\left(1-\frac{1}{4}(|0.921+0.866|^2+|0.390+0.500|^2)\right)$$
$$= 0.552$$

χ is to measure the top half of the column vector: top 8 states

- $\chi = 4$ because should only be four numerical outputs

Step D α_{final}
0
$\frac{1}{\sqrt{8\chi}}(0.921+0.866)$
0
$\frac{1}{\sqrt{8\chi}}(0.390+0.500)$
$\frac{1}{\sqrt{8\chi}}(0.141+0.866)$
0
$\frac{1}{\sqrt{8\chi}}(0.990+0.500)$
0
0
0
0
0
0
0
0
0

Summary

Key Points:

- Renormalization is what allows distance to be calculated
- Hadamard Gate is data classifier which is needed for the conditional measurement
- The combination of the conditional measurement with renormalization allows us to get back to $1-d^2$

Further Work:

- Look at other points on the Titanic data set to see how accurate the classifier is through work with the Simulator and NumPy